Solution by H. C. FEEMSTER, York College, York, Nebraska.

$$\sum_{n=1}^{n=\infty} n^2 x^{n-1} = 1 + 4x + 9x^2 + 16x^3 + \ldots + r^2 x^{r-1} + (r+1)^2 x^r + (r+2)^2 x^{r+1} + \ldots.$$

and $(1-x)^3 \sum_{n=1}^{n=\infty} n^2 x^{n-1} = [1 + 4x + 9x^2 + 16x^3 + \ldots + r^2 x^{r-1} + (r+1)^2 x^r$

$\qquad + (r+2)^2 x^{r+1} + \ldots] - [3x + 12x^2 + 27x^3 + \ldots + 3(r-1)^2 x^{r-1} + 3r^2 x^r$

$\qquad + 3(r+1)^2 x^{r+1} + \ldots + [3x^2 + 12x^3 + \ldots + 3(r-2)^2 x^{r-1} + 3(r-1)^2 x^r$

$\qquad + 3r^2 x^{r+1} + \ldots - [x^3 + \ldots + (r-3)^2 x^{r-1} + (r-2)^2 x^r + (r-1)^2 x^{r+1} + \ldots]$

$\qquad = 1 + x$, so

$$\therefore \sum_{n=1}^{n=\infty} n^2 x^{n-1} = \frac{1+x}{(1-x)^3}, \text{ as required.}$$

Also solved by H. Prime, J. Scheffer, A. M. Harding, and Elijah Swift.

373. **Proposed by X, National Electric Light Association, Brooklyn, N. Y.**

(a) For underground distribution of direct current electrical energy, we have $DA^n - CB^n = H$, where the only unknown is n, which represents the number of years it will take a large direct current low tension feeder to pay by line loss saving for the increased investment over a smaller feeder.

(b) $bVf^2l^2\beta^2 + aVf\beta^{1.6} = W$, the iron loss equation which is to be solved for β. When $bVf^2l^2\beta^2$ represents the eddy current loss in the core of a transformer, and $aVf\beta^{1.6}$ is the hysteresis loss in the core. a, b, and l are constants of the core, V is the voltage, f is the frequency, and β is the flux density and is the only unknown in the equation.

Letting $bVf^2l^2 = A$, and $aVf = C$, we have $A\beta^2 + C\beta^{1.6} = W$.

Solution by E. B. ESCOTT, Ann Arbor, Michigan.

(a) $DA^n - CB^n = H$, n unknown. Since $A = 10^{\log A}$ and $B = 10^{\log B}$, the equation may be written

$$D.10^{n\log A} - C.10^{n\log B} = H.$$

Putting $10^n = x$, the equation becomes $Dx^{\log A} - Cx^{\log B} = H$.

This equation and the one under

(b) $$A\beta^2 + C\beta^{1.6} = W$$

are trinomial equations, which may be solved in various ways. One of the simplest methods to use in practice is by addition and subtraction of logarithms. This is given in C. Runge, *Praxis der Gleichungen* (Leipzig, 1900), pages 140-157.

Gundelfinger has given tables to three decimals which enable one to

solve trinomial equations of all degrees with any coefficients.   S. Gundelfinger, *Tafeln zur Berechnung der reellen Wurzeln sämtlicher trinomischer Gleichungen* (Leipzig, 1897).

For references to the literature of trinomial equations, see the article, R. Mehmke, *Calculs numériques*, in the *Encyclopédie des Sciences Mathématiques*, tome I, Volume 4, I 23.

In § 41, pages 320-325, the following method is given.

In $A \beta^2 + C \beta^{1.6} = W$, put $\beta = kx^n$.   Then $Ak^2x^{2n} + Ck^{1.6}x^{1.6n} = W$.

Let $1.6n = 1$, whence $n = \dfrac{1}{1.6} = .625$.

Let $Ak^2 = Ck^{1.6}$, whence $Ak^{0.4} = C$, and $k = \left(\dfrac{C}{A}\right)^{2.5}$.

Then equation reduces to $x^{1.25} + x = D$.

Make a table of values of $x^{1.25} + x$ for different values of x with as small an interval as desired.   Then any given equation may be solved by simple interpolation.

For graphic solution (by nomography), see Articles 45, 51, 61; also M. d'Ocagne, *Traité de Nomographie* (Paris, 1899), pages 367, 387.

Another method is given by F. Schlepps, *Euber die Auflösung trinomischer Gleichungen aller Grade* (Halle a. S. (1899), 15 pages).   [See *Jahrbuch ü. d. Fortschritté der Math.*, Vol. 30, page 104.]

GEOMETRY.

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400.   Proposed by FRANCIS RUST, C. E., Pittsburgh, Pennsylvania.

Given a circle and a point P without; construct, *using the straight edge only*, the two tangents to the circle through P.

Solution by H. E. TREFETHEN, Colby College, Waterville, Maine; H. C. FEEMSTER, York College, York, Nebraska, and ELMER SCHUYLER, New York City.

From P draw two secants and complete the inscribed quadrilateral thus determined.   Let PQ be the external diagonal and R the point of intersection of the internal diagonals.   Hence each side of the triangle PQR is the polar of the opposite vertex and QR cuts the circle in S and T, the points of contact of the required tangents PS and PT.   Thus the construction is effected with the ruler only.

Also solved by M. E. Graber and E. B. Escott.

401.   Proposed by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

Find by Euclidean geometry a point whose distances from the vertices of an equilateral triangle are in the ratio 3:4:5. The general case of ratio $a:b:c$ would prove interesting.

I.   Solution by W. J. GREENSTREET, M. A., Editor The Mathematical Gazette, Burgfield, England.

Let ABC be any triangle.   The locus of a point P such that $PA:PB = x:y$ is well known to be a circle.   Let the circular loci of the points P given